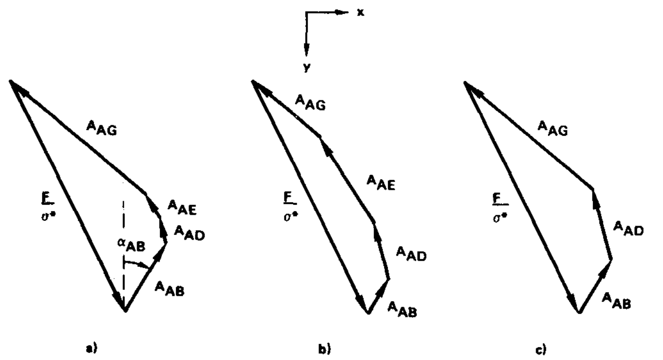

Fig. 4 Typical solutions of Eq. (4) for cross-sectional areas of truss in Fig. 2 which is subjected to force F.

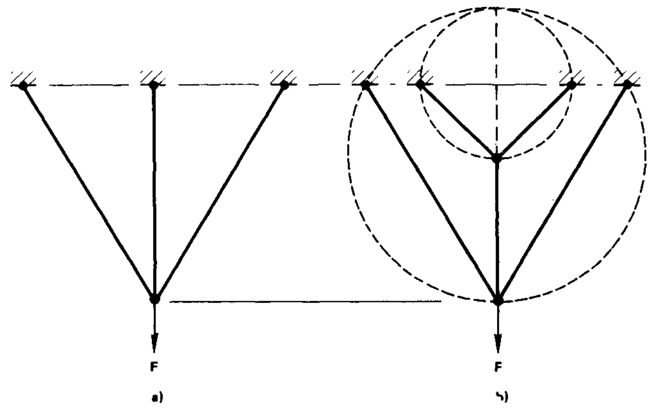

Fig. 6 a) Indeterminate truss; b) converted to fully stressed truss.

the volume of the truss in Fig. 3, since F_y may be arbitrarily apportioned between the tension and compression trusses.

Suppose, now, that the length of the component in Fig. 1 is less than the value required by Eq. (2), but that the location of point A and angle α_{AB} are unchanged. This implies that the other end point, now denoted by B', lies within the circle of diameter l_o and, from Eq. (1), that component AB' will be fully stressed for a deflection δ_A only if

$$(\delta_{B'})\cos\beta_{AB'} = \delta_A \cos\alpha_{AB} - \varepsilon^* l_{AB'} \tag{8}$$

where $\delta_{B'}$ and $\beta_{AB'}$ are values at end B' of component AB', and $l_{AB'} < l_{AB}$. Since the terms on the right-hand side of Eq. (8) are all specified, any convenient combination of $\delta_{B'}$ and $\beta_{AB'}$ which satisfies Eq. (8) may be selected, and point B' may then be treated as the free node of a second "imbedded" circle-chord truss which is to be fully stressed for the deflection $\delta_{B'}$. A resulting fully stressed truss, derived from Fig. 2, is shown in Fig. 5. Note that the direction of AB' must lie within the sector bounded by the extreme members of the second truss in order for the second truss to equilibrate force $F_{AB'}$. Also, from Eq. (2)

$$l_o' = \delta_{B'}/\varepsilon^* \tag{9}$$

Regardless of the magnitude of l_o', the circle of this diameter must intersect the original circle at point B. This may be shown as follows: assume the extension of line AB' intersects the circle of diameter l_o' at a point B''. Then, from Eqs. (2) and (8)

$$l_{B'B''} = \frac{\delta_{B'}}{\varepsilon^*}\cos\beta_{AB'} = \frac{\cos\beta_{AB'}}{\varepsilon^*}\left(\frac{\delta_A \cos\alpha_{AB} - \varepsilon^* l_{AB'}}{\cos\beta_{AB'}}\right)$$
$$= (\delta_A/\varepsilon^*)\cos\alpha_{AB} - l_{AB'} = l_{AB} - l_{AB'} = l_{B'B}$$

Therefore $B'' \equiv B$.

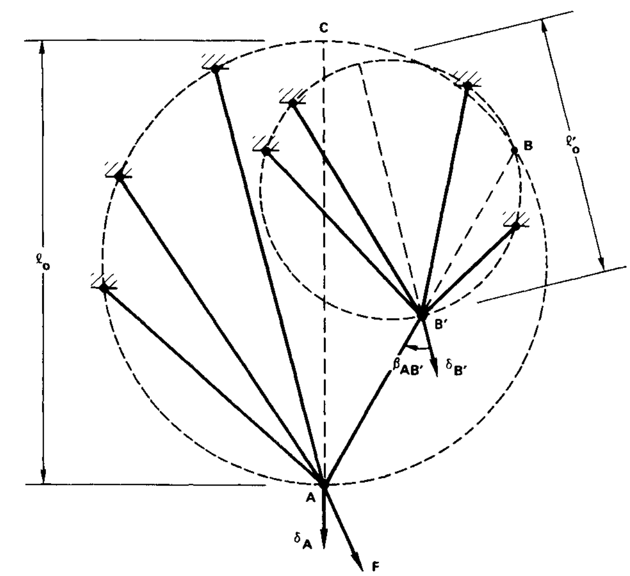

Fig. 5 Imbedded circle-chord truss.

Since $F_{AB'} = F_{AB} = \sigma^* A_{AB}$, it follows from Fig. 5, and Eqs. (7–9) and (2) that the volume of the second truss is given by

$$V' = A_{AB}(l_{AB} - l_{AB'}) \tag{10}$$

i.e., the volume of the imbedded fully stressed truss equals the volume of the missing part ($B'B$) of the primary truss.

This concept of imbedded circle-chord trusses is particularly useful in providing alternative locations for support points. For example, the truss of Fig. 6a, which cannot be fully stressed without deleting one component, may be replaced by the fully stressed design in Fig. 6b. The truss in Fig. 6b has all support points located along the same horizontal line as in Fig. 6a.

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Orthogonality Procedure for Forced Response of Multispan Beams

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Introduction

THE orthogonality property of the free vibration modes of classical beam theory are well known as evidenced by the concise treatment given by Meirovitch.[1] Because of the generality of the classical formulation of the orthogonality principle, the dynamic response of beams of constant as well as variable (EI) can be treated by the usual Sturm-Liouville (normal mode) procedure.[1] In the case of beam systems consisting of several intermediate spring supports along with possible discontinuities in (EI), the use of the classical form of orthogonality relation to develop the dynamic solution becomes awkward. With this in mind, the present Note develops a more general form of orthogonality relation. In particular, for a beam system consisting of L discrete spans with several intermediate spring-type supports, the governing beam equation for a given subspan is

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no longer self-adjoint. Hence, the traditional orthogonality relation does not apply.[1,2] To circumvent this difficulty, a piecewise-weighted orthogonality procedure[2] is developed herein which can handle the abovementioned problem. Based on the piecewise-weighted orthogonality procedure, the general solution is developed for a beam system consisting of discrete spans with intermediate spring-type supports subjected to time-dependent body forces, and support loads.

Equations

For a beam system consisting of several intermediate spring or hinge supports along with possibly distinct spans, the governing field equations and their associated end and interbeam conditions are defined by

$$[(EI)^{(l)}w^{(l)}_{,xx}]_{,xx}-F^{(l)}_{(x,t)}=-\rho^{(l)}w^{(l)}_{,tt};\quad \begin{cases}x\,\varepsilon\,[x_l,x_{l+1}]\\ l=1,2,3,\ldots,L\end{cases} \tag{1}$$

at $x=$ end

$$\left.\begin{aligned}[(EI)^{(k)}w^{(k)}_{,xx}]_{,x}\mp\kappa_{1l}w^{(k)}&=\tilde V_l\\ (EI)^{(k)}w^{(k)}_{,xx}\pm\kappa_{2l}w^{(k)}_{,x}&=\tilde M_l\end{aligned}\right\}k=1,L;\ l=1,L+1 \tag{2}$$

at $x=x_{l+1}$, $l=1,2,3,\ldots,L-1$

$$\begin{aligned}&w^{(l)}=w^{(l+1)}\quad w^{(l)}_{,x}=w^{(l+1)}_{,x}\\ &[(EI)^{(l)}w^{(l)}_{,xx}-(EI)^{(l+1)}w^{(l+1)}_{,xx}]_{,x}-\kappa_{1l+1}w^{(l)}=\tilde V_{l+1}\\ &(EI)^{(l)}w^{(l)}_{,xx}-(EI)^{(l+1)}w^{(l+1)}_{,xx}+\kappa_{2l+1}w^{(l)}_{,x}=\tilde M_{l+1}\end{aligned} \tag{3}$$

where $E^{(l)}$, $I^{(l)}$, $w^{(l)}$, $F^{(l)}$, and $\rho^{(l)}$ are, respectively, the Young's modulus, moment of inertia, lateral displacement, lateral body force, and density of the lth segment. Furthermore, κ_{1l}, κ_{2l}, $\tilde V_l$, and $\tilde M_l$ represent the lateral and rotational spring constants and the time dependent shear and moment loadings of the lth joint, respectively. Finally L denotes the sum total of the number of supports and distinct segments.

The time-dependent end and interbeam conditions, Eqs. (2) and (3), can be homogenized through the modified use of the classical Mindlin-Goodman procedure.[3] Hence by letting

$$w^{(l)}=W^{(l)}+\sum_{n=1}^{2L+2}\sum_{m=1}^{4}C_{mnl}\,g_{ml}\,h_n \tag{4}$$

where

$$g_{1l}=1,\qquad g_{2l}=x$$

$$\langle g_{3l},g_{4l}\rangle=\int^x\int^u\frac{1}{[E(v)I(v)]^{(l)}}\langle 1,v\rangle\,dv\,du \tag{5}$$

$$\left.\begin{aligned}h_{2n-1}&=\tilde V_n\\ h_{2n}&=\tilde M_n\end{aligned}\right\}n=1,2,\ldots,L+1 \tag{6}$$

the proper choice of C_{mnl} reduces Eqs. (1–3) to

$$[(EI)^{(l)}W^{(l)}_{,xx}]_{,xx}-F^{(l)}_{(x,t)}=-\rho^{(l)}W^{(l)}_{,tt}-\rho^{(l)}\sum_{n=1}^{2L+2}\sum_{m=1}^{4}C_{mnl}\,g_{ml}\,h_{n,tt}\begin{cases}x\,\varepsilon\,[x_l,x_{l+1}]\\ l=1,2,\ldots,L\end{cases} \tag{7}$$

at $x=$ end

$$\left.\begin{aligned}[(EI)^{(k)}W^{(k)}_{xx}]_{,x}\mp\kappa_{1l}W^{(k)}&=0\\ (EI)^{(k)}W^{(k)}_{,xx}\pm\kappa_{2l}W^{(k)}_{,x}&=0\end{aligned}\right\}k=1,L;\ l=1,L+1 \tag{8}$$

at $x=x_{l+1}$, $l=1,2,\ldots,L-1$

$$\begin{aligned}&W^{(l)}=W^{(l+1)},\qquad W^{(l)}_{,x}=W^{(l+1)}_{,x}\\ &[(EI)^{(l)}W^{(l)}_{,xx}-(EI)^{(l+1)}W^{(l+1)}_{,xx}]_{,x}-\kappa_{1l+1}W^{(l)}=0\\ &(EI)^{(l)}W^{(l)}_{,xx}-(EI)^{(l+1)}W^{(l+1)}_{,xx}+\kappa_{2l+1}W^{(l)}_{,x}=0\end{aligned} \tag{9}$$

Separation of Variables

Assuming that formal separation of variables is possible $w^{(l)}$ takes the form

$$W^{(l)}_{(x,t)}=\sum_{p=1}^{\infty}\xi^{(l)}_p(x)\zeta^{(l)}_p(t) \tag{10}$$

Because of the form of the interbeam conditions, Eqs. (9), in terms of Eq. (10), it follows that

$$\zeta^{(l)}_p=\zeta_p;\quad l=1,2,\ldots,L \tag{11}$$

Furthermore, since $\zeta_p\propto e^{\Omega_p{}^{(l)}t}$ for the free vibration case, because of Eq. (11), $\Omega^{(l)}_p=\Omega_p$ for all $l\,\varepsilon\,[1,L]$. Hence Eq. (10) reduces to

$$W^{(l)}=\sum_{p=1}^{\infty}\xi^{(l)}_p\zeta_p \tag{12}$$

In terms of Eq. (12), it follows that $\xi^{(l)}$ satisfies the following eigenvalue problem

$$[(EI)^{(l)}\xi^{(l)}_{,xx}]_{,xx}=\rho^{(l)}\Omega^2\xi^{(l)}\begin{cases}x\,\varepsilon\,[x_l,x_{l+1}]\\ l=1,2,\ldots,L\end{cases} \tag{13}$$

at $x=$ end

$$\left.\begin{aligned}[(EI)^{(k)}\xi^{(k)}_{,xx}]_{,x}\mp\kappa_{1l}\xi^{(k)}&=0\\ (EI)^{(k)}\xi^{(k)}_{,xx}\pm\kappa_{2l}\xi^{(k)}_{,x}&=0\end{aligned}\right\}k=1,L;\ l=1,L+1 \tag{14}$$

at $x=x_{l+1}$, $l=1,2,\ldots,L-1$

$$\begin{aligned}&\xi^{(l)}=\xi^{(l+1)},\qquad \xi^{(l)}_{,x}=\xi^{(l+1)}_{,x}\\ &[(EI)^{(l)}\xi^{(l)}_{,xx}-(EI)^{(l+1)}\xi^{(l+1)}_{,xx}]_{,x}-\kappa_{1l+1}\xi^{(l)}=0\\ &(EI)^{(l)}\xi^{(l)}_{,xx}-(EI)^{(l+1)}\xi^{(l+1)}_{,xx}+\kappa_{2l+1}\xi^{(l)}_{,x}=0\end{aligned} \tag{15}$$

To establish the orthogonality of the eigenfunction set $\xi^{(l)}$, assume that for distinct eigenvalues Ω_p and Ω_q

$$\begin{aligned}[(EI)^{(l)}\xi^{(l)}_{p,xx}]_{,xx}&=\rho^{(l)}\Omega_p^2\xi^{(l)}_p\\ [(EI)^{(l)}\xi^{(l)}_{q,xx}]_{,xx}&=\rho^{(l)}\Omega_q^2\xi^{(l)}_q\end{aligned}\begin{cases}x\,\varepsilon\,[x_l,x_{l+1}]\\ l=1,2,\ldots,L\end{cases} \tag{16}$$

After several standard manipulations, Eqs. (16) reduces to

$$(\Omega_p^2-\Omega_q^2)\rho^{(l)}\xi^{(l)}_p\xi^{(l)}_q=\xi^{(l)}_q[(EI)^{(l)}\xi^{(l)}_{p,xx}]_{,xx}-\xi^{(l)}_p[(EI)^{(l)}\xi^{(l)}_{q,xx}]_{,xx} \tag{17}$$

Integrating Eq. (17) over the intervals $x\,\varepsilon\,[x_l,x_{l+1}]$ it follows that

$$(\Omega_p^2-\Omega_q^2)\int_{x_l}^{x_{l+1}}\rho^{(l)}\xi^{(l)}_p\xi^{(l)}_q\,dx=\{\xi^{(l)}_q[(EI)^{(l)}\xi^{(l)}_{p,xx}]_{,x}-\xi^{(l)}_p[(EI)^{(l)}\xi^{(l)}_{q,xx}]_{,x}+\xi^{(l)}_{p,x}(EI)^{(l)}\xi^{(l)}_{q,xx}-\xi^{(l)}_{q,x}(EI)^{(l)}\xi^{(l)}_{p,xx}\}\Big|_{x_l}^{x_{l+1}} \tag{18}$$

Since the right-hand side of Eq. (18) is nonzero, Eq. (13) are nonself-adjoint in the traditional sense.[1,2] Applying several manipulations to Eqs. (15), the following interbeam identities can be developed

$$\begin{aligned}&\xi^{(l)}_q[(EI)^{(l)}\xi^{(l)}_{p,xx}]_{,x}-\xi^{(l)}_p[(EI)^{(l)}\xi^{(l)}_{q,xx}]_{,x}+\xi^{(l)}_{p,x}(EI)^{(l)}\xi^{(l)}_{q,xx}-\xi^{(l)}_{q,x}(EI)^{(l)}\xi^{(l)}_{p,xx}=\\ &\xi^{(l+1)}_q[(EI)^{(l+1)}\xi^{(l+1)}_{p,xx}]_{,x}-\xi^{(l+1)}_p[(EI)^{(l+1)}\xi^{(l+1)}_{q,xx}]_{,x}+\xi^{(l+1)}_{p,x}(EI)^{(l+1)}\xi^{(l+1)}_{q,xx}-\xi^{(l+1)}_{q,x}(EI)^{(l+1)}\xi^{(l+1)}_{p,xx}\end{aligned} \tag{19}$$

Using Eqs. (14) and (19) in conjunction with Eq. (18), it follows that

$$(\Omega_p^2-\Omega_q^2)\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}}\rho^{(l)}\xi^{(l)}_p\xi^{(l)}_q\,dx=\begin{cases}0;\ p\neq q\\ \neq 0;\ p=q\end{cases} \tag{20}$$

Hence, rather than being self-adjoint in the local sense, Eqs. (13–15) are self-adjoint in the global sense described by the piecewise weighted orthogonality principle given by Eq. (20).

Solution

That $\xi^{(l)}_p$ form a complete set can be established through the use of standard variational procedures.[4] Thus if $F^{(l)}$, g_{ml} and the initial conditions $w^{(l)}(x,0)$ and $w^{(l)}(x,0)_{,t}$ all satisfy Dirichlet's conditions, the following formal expansions can be taken:

$$(1/\rho^{(l)})F^{(l)}(x,t)=\sum_{p=1}^{\infty}\xi^{(l)}_pf_p \tag{21}$$

$$\sum_{m=1}^{4}C_{mnl}\,g_{ml}=\sum_{p=1}^{\infty}\xi^{(l)}_pg_{np} \tag{22}$$

$$w^{(l)}(x,0)=\sum_{p=1}^{\infty}\xi^{(l)}_pw_{p0} \tag{23}$$

$$w^{(l)}(x,0)_{,t}=\sum_{p=1}^{\infty}\xi^{(l)}_pw_{p1} \tag{24}$$

where

$$\langle f_p, g_{np}, w_{p0}, w_{p1} \rangle =$$
$$\frac{1}{N_p}\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} \left\langle (1/\rho^{(l)})F^{(l)}, \sum_{m=1}^{4} C_{mnl}\, g_{ml}, w^{(l)}(0), w^{(l)}(0)_{,t} \right\rangle \rho^{(l)}\xi_p^{(l)}\, dx \tag{25}$$

such that

$$N_p = \sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} \rho^{(l)}\xi_p^{(l)}\xi_p^{(l)}\, dx \tag{26}$$

Substituting Eqs. (10, 21, and 22) into Eq. (7) gives

$$\sum_{p=1}^{\infty}\Big\{[(EI)^{(l)}\xi_{p,xx}^{(l)}\zeta_p]_{,xx} - \xi_p^{(l)} f_p + \rho^{(l)}\xi_p^{(l)}\zeta_{p,tt} +$$
$$\rho^{(l)}\sum_{n=1}^{2L+2} g_{np}h_{n,tt}\,\xi_p^{(l)}\Big\} = 0;\ l = 1,2,\ldots,L \tag{27}$$

Now applying the piecewise-weighted orthogonality principle, Eq. (20), yields

$$\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} \text{Eqs. (27)}\ \ \xi_p^{(l)}\, dx \to \zeta_{p,tt} + \Omega_p^2\zeta_p = f_p - \sum_{n=1}^{2L+2} g_{np}h_{n,tt} \tag{28}$$

In terms of the reduced initial conditions, Eq. (25), the solution of Eq. (28) takes the form

$$\zeta_{p(t)} = w_{p0}\cos\Omega_p t + \frac{w_{p1}}{\Omega_p}\sin\Omega_p t +$$
$$\frac{1}{\Omega_p}\int_0^t \left[f_p(\tau) - \sum_{n=1}^{2L+2} g_{np}h_n(\tau)_{,\tau\tau}\right]\sin\Omega_p(t-\tau)\, d\tau \tag{29}$$

Hence $w^{(l)}$ is finally given by

$$w^{(l)}(x,t) = \sum_{p=1}^{\infty}\xi_p^{(l)}\zeta_p + \sum_{n=1}^{2L+2}\sum_{m=1}^{4} C_{mnl}\, g_{ml}\, h_n \tag{30}$$

Discussion

Because of the form of the eigenfunction expansion given by Eq. (10), the solution of Eqs. (1–3) has been reduced basically to the problem of obtaining the eigenvalues of Eqs. (13–15). To simplify the search for the eigenvalues of these equations, the realness and positive definiteness of Ω^2 will be established.

To prove the realness of Ω^2 assume that

$$\begin{aligned}\xi^{(l)} &= Re\{\xi^{(l)}\} + jIm\{\xi^{(l)}\}\\ \Omega^2 &= Re\{\Omega^2\} + jIm\{\Omega^2\}\end{aligned} \tag{31}$$

where $j = (-1)^{1/2}$. In the spirit of the development of the piecewise-weighted orthogonality procedure, Eq. (20), it can be shown that in terms of Eqs. (31), Eq. (13) can be directly manipulated to yield

$$Im\{\Omega^2\}\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} \rho^{(l)}(Re^2\{\xi^{(l)}\} + Im^2\{\xi^{(l)}\})\, dx =$$
$$\sum_{l=1}^{L}[Re\{\xi^{(l)}\}((EI)^{(l)}Im\{\xi^{(l)}\}_{,xx})_{,x} -$$
$$Im\{\xi^{(l)}\}((EI)^{(l)}Re\{\xi^{(l)}\}_{,xx})_{,x} +$$
$$Im\{\xi^{(l)}\}_{,x}(EI)^{(l)}Re\{\xi^{(l)}\}_{,xx} -$$
$$Re\{\xi^{(l)}\}_{,x}(EI)^{(l)}Im\{\xi^{(l)}\}_{,xx}]\Big|_{x_l}^{x_{l+1}} \tag{32}$$

Furthermore, using Eqs. (31) in conjunction with the interbeam conditions, Eqs. (14), the following identities can be established:

$$Re\{\xi^{(l)}\}[(EI)^{(l)}Im\{\xi^{(l)}\}_{,xx}]_{,x} - Im\{\xi^{(l)}\}[(EI)^{(l)}Re\{\xi^{(l)}\}_{,xx}]_{,x} +$$
$$Im\{\xi^{(l)}\}_{,x}(EI)^{(l)}Re\{\xi^{(l)}\}_{,xx} - Re\{\xi^{(l)}\}_{,x}(EI)^{(l)}Im\{\xi^{(l)}\}_{,xx} =$$
$$Re\{\xi^{(l+1)}\}[(EI)^{(l+1)}Im\{\xi^{(l+1)}\}_{,xx}]_{,x} -$$
$$Im\{\xi^{(l+1)}\}[(EI)^{(l+1)}Re\{\xi^{(l+1)}\}_{,xx}]_{,x} +$$
$$Im\{\xi^{(l+1)}\}_{,x}(EI)^{(l+1)}Re\{\xi^{(l+1)}\}_{,xx} -$$
$$Re\{\xi^{(l+1)}\}_{,x}(EI)^{(l+1)}Im\{\xi^{(l+1)}\}_{,xx} \tag{33}$$

Substituting Eqs. (14) and (33) into Eq. (32) yields

$$Im\{\Omega^2\}\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} \rho^{(l)}(Re^2\{\xi^{(l)}\} + Im^2\{\xi^{(l)}\})\, dx \equiv 0 \tag{34}$$

Since all the integrals appearing in Eq. (34) are positive definite, it follows that $Im\{\Omega^2\} \equiv 0$, thus Ω^2 is purely real.

To establish that Ω^2 is positive definite, after several manipulations, the variational formulation of Eqs. (14) and (15) can be used to develop the following form of Rayleigh's quotient

$$\Omega^2 = \frac{\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} (EI)^{(l)}\xi_{,xx}^{(l)}\xi_{,xx}^{(l)}\, dx}{\sum_{l=1}^{L}\int_{x_l}^{x_{l+1}} \rho^{(l)}\xi^{(l)}\xi^{(l)}\, dx} \tag{35}$$

Since both the numerator and denominator of Eq. (35) are positive definite, $\Omega^2 > 0$.

Because of the generality of the procedure developed herein, branched beam systems can also be handled. This is possible through the straight forward modification of the piecewise-weighted orthogonality relation depicted by Eq. (19) along with the inclusion of torsional effects. In fact the orthogonalization procedure can also be extended to Timoshenko-type theory as well as to composite plate and shell configurations with and without branches.

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Oblique Compressible Sears Function

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Introduction

THIS Note concerns the lift response of a thin, infinite-span wing flying subsonically through a stationary sinusoidal gust at an arbitrary angle to the lines of constant phase: that is, the generalization of Sears' classical result[1] to oblique gusts and compressible flow.

With the freestream U in the positive x direction and the wing of chord $2b$ along the y axis, the upwash on the wing is

$$w(x,y,t) = w_o\exp\{-i[k_1(x-Ut)/b + k_2 y/b]\},\quad |x| \leq b \tag{1}$$

The corresponding lift distribution is governed by three nondimensional parameters: chordwise wavenumber (or reduced frequency) k_1, spanwise wavenumber k_2, and Mach number M (w_o/U, small by hypothesis, is unimportant as the solution is linear therein). We *assume* that the usual linearized equation for the velocity potential is valid for all combinations of parameters of interest.

Sears' solution (for $k_2 = M = 0$) was extended by Osborne[2] to

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